

# Self-induced spectral splits in supernova neutrino fluxes

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In the dense-neutrino region above the neutrino sphere of a supernova ( $r \lesssim 400$  km), neutrino-neutrino refraction causes collective flavor transformations. They can lead to “spectral splits” where an energy  $E_{\text{split}}$  splits the transformed spectrum sharply into parts of almost pure but different flavors. Unless there is an ordinary MSW resonance in the dense-neutrino region,  $E_{\text{split}}$  is determined by flavor-lepton number conservation alone. Spectral splits are created by an adiabatic transition between regions of large and small neutrino density. We solve the equations of motion in the adiabatic limit explicitly and provide analytic expressions for a generic example.

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## I. INTRODUCTION

At large densities, neutrino-neutrino refraction causes nonlinear flavor oscillation phenomena with sometimes perplexing results [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. In the region between the neutrino sphere and a radius of about 400 km in core-collapse supernovae (SNe), the neutrino flavor content evolves dramatically [7, 8, 9, 10, 11, 12, 13]. The global features of this self-induced transformation are equivalent to the motion of a gyroscopic pendulum in flavor space [11, 12]. However, this picture does not explain the “spectral splits” that have been numerically observed in the transformed fluxes [9, 10, 13]. In a typical case, the primary  $\nu_e$  flux below a split energy  $E_{\text{split}}$  emerges from the dense-neutrino region in its original flavor, whereas above  $E_{\text{split}}$ , it is completely transformed to  $\nu_x$  (some mixture of  $\nu_\mu$  and  $\nu_\tau$ ), the step at  $E_{\text{split}}$  being very sharp. (To be specific we explore the  $\nu_e$ - $\nu_x$  system with the atmospheric  $\Delta m^2$  and the small 13-mixing angle.)

It has been suggested that an adiabatic transition from high to low neutrino density is the primary cause for the split [9, 12]. Dense neutrinos perform synchronized oscillations: all modes oscillate with a common frequency  $\omega_{\text{synch}}$ , even though their individual frequencies vary as  $\omega = |\Delta m^2/2E|$ . Flavor oscillations can be visualized as the precession of polarization vectors  $\mathbf{P}_\omega$  in a “flavor  $\mathbf{B}$  field.” The  $\mathbf{P}_\omega$  “stick together” by the  $\nu$ - $\nu$ -interaction, thus forming a collective object that precesses around  $\mathbf{B}$ . The collectivity is lost when the neutrino density decreases. However, if the decrease is slow, all  $\mathbf{P}_\omega$  align themselves with or against  $\mathbf{B}$  in the process of decoupling from each other. Eventually they all precess with their individual  $\omega$  around  $\mathbf{B}$ , but without visible consequences because of their (anti-)alignment with  $\mathbf{B}$ .

We extend this interpretation of the split phenomenon in several ways. We (i) show that flavor-lepton number conservation determines  $E_{\text{split}}$ , (ii) solve the equations of motion explicitly in the adiabatic limit, and (iii) provide an analytic result for a generic case.

## II. EQUATIONS OF MOTION

We represent the flavor content of an isotropic  $\nu$ - $\bar{\nu}$  gas by flavor polarization vectors  $\mathbf{P}_\omega$  and  $\bar{\mathbf{P}}_\omega$ , where overbarred quantities correspond to  $\bar{\nu}$ . We define their global counterparts as  $\mathbf{P} = \int_0^\infty d\omega \mathbf{P}_\omega$  and  $\bar{\mathbf{P}} = \int_0^\infty d\omega \bar{\mathbf{P}}_\omega$  and introduce  $\mathbf{D} \equiv \mathbf{P} - \bar{\mathbf{P}}$ , representing the net lepton number. The equations of motion (EOMs) are [11, 16]

$$\partial_t \mathbf{P}_\omega = (\omega \mathbf{B} + \lambda \mathbf{L} + \mu \mathbf{D}) \times \mathbf{P}_\omega \quad (1)$$

and the same for  $\bar{\mathbf{P}}_\omega$  with  $\omega \rightarrow -\omega$ . Here  $\lambda \equiv \sqrt{2} G_F n_e$  represents the usual matter potential and  $\mu \equiv \sqrt{2} G_F n_\nu$  the  $\nu$ - $\nu$  interaction strength, where  $n_e$  and  $n_\nu$  are the electron and neutrino densities. We work in the mass basis where  $\mathbf{B} = (0, 0, -1)$  corresponds to the normal and  $\mathbf{B} = (0, 0, +1)$  to the inverted mass hierarchies. The interaction direction  $\mathbf{L}$  is a unit vector such that  $\mathbf{B} \cdot \mathbf{L} = \cos 2\theta$  with  $\theta$  being the vacuum mixing angle. Unless there is an MSW resonance in the dense-neutrino region, one can eliminate  $\lambda \mathbf{L}$  from Eq. (1) by going into a rotating frame, at the expense of a small effective mixing angle [8, 11]. The only difference for antineutrinos is that in vacuum they oscillate “the other way round.” Therefore, instead of using  $\bar{\mathbf{P}}_\omega$  we may extend  $\mathbf{P}_\omega$  to negative frequencies such that  $\bar{\mathbf{P}}_\omega = \mathbf{P}_{-\omega}$  ( $\omega > 0$ ) and use only  $\mathbf{P}_\omega$  with  $-\infty < \omega < +\infty$ . In these terms,  $\mathbf{D} = \int_{-\infty}^{+\infty} d\omega s_\omega \mathbf{P}_\omega$ , where  $s_\omega \equiv \text{sign}(\omega) = \omega/|\omega|$ .

After elimination of  $\lambda \mathbf{L}$ , the EOM for  $\mathbf{D}$  can be obtained by integrating Eq. (1) with  $s_\omega$ :

$$\partial_t \mathbf{D} = \mathbf{B} \times \mathbf{M} \quad \text{where} \quad \mathbf{M} \equiv \int_{-\infty}^{+\infty} d\omega s_\omega \omega \mathbf{P}_\omega. \quad (2)$$

It shows that  $\partial_t (\mathbf{D} \cdot \mathbf{B}) = 0$  so that  $D_z = \mathbf{B} \cdot \mathbf{D}$  is conserved [11]. The in-medium mixing angle above a SN core is small and therefore the mass and interaction basis almost coincide. Collective effects then only induce pair transformations of the form  $\nu_e \bar{\nu}_e \rightarrow \nu_x \bar{\nu}_x$ , whereas the excess  $\nu_e$  flux from deleptonization is conserved.

### III. ADIABATIC SOLUTION

We rewrite the EOMs in terms of an “effective Hamiltonian” for the individual modes as

$$\partial_t \mathbf{P}_\omega = \mathbf{H}_\omega \times \mathbf{P}_\omega \quad \text{where} \quad \mathbf{H}_\omega = \omega \mathbf{B} + \mu \mathbf{D}. \quad (3)$$

In the adiabatic limit each  $\mathbf{H}_\omega$  moves slowly compared to the precession of  $\mathbf{P}_\omega$  so that the latter follows the former. We assume that initially all  $\mathbf{P}_\omega$  represent the same flavor and thus are aligned. If initially  $\mu$  is large, every  $\mathbf{P}_\omega$  is practically aligned with  $\mathbf{H}_\omega$ . Therefore, in the adiabatic limit it stays aligned with  $\mathbf{H}_\omega$  for the entire evolution:

$$\mathbf{P}_\omega(\mu) = \hat{\mathbf{H}}_\omega(\mu) P_\omega, \quad (4)$$

which solves the EOMs. Here  $P_\omega \equiv |\mathbf{P}_\omega|$  and  $\hat{\mathbf{H}}_\omega \equiv \mathbf{H}_\omega/|\mathbf{H}_\omega|$  is a unit vector. Here and henceforth we assume an excess flux of neutrinos over antineutrinos, implying that initially  $\mathbf{P}_\omega$  and  $\mathbf{D}$  are collinear and  $D_z > 0$ .

According to Eq. (3) all  $\mathbf{H}_\omega$  lie in the plane spanned by  $\mathbf{B}$  and  $\mathbf{D}$  which we call the “co-rotating plane.” In the adiabatic limit all  $\mathbf{P}_\omega$ , and consequently  $\mathbf{M}$ , also stay in that plane. Therefore we can decompose

$$\mathbf{M} = b \mathbf{B} + \omega_c \mathbf{D} \quad (5)$$

and rewrite the EOM of Eq. (2) as

$$\partial_t \mathbf{D} = \omega_c \mathbf{B} \times \mathbf{D}. \quad (6)$$

Therefore  $\mathbf{D}$  and the co-rotating plane precess around  $\mathbf{B}$  with the common or “co-rotation frequency”  $\omega_c$ .

We conclude that the system evolves simultaneously in two ways: a fast precession around  $\mathbf{B}$  determined by  $\omega_c = \omega_c(\mu)$  and a drift in the co-rotating plane caused by the explicit  $\mu(t)$  variation. To isolate the latter from the former, we go (following Ref. [8]) into the co-rotating frame where the individual Hamiltonians become

$$\mathbf{H}_\omega = (\omega - \omega_c) \mathbf{B} + \mu \mathbf{D}. \quad (7)$$

We use the same notation because the relevant components  $H_{\omega z}$ ,  $H_{\omega \perp}$ ,  $D_z$ , and  $D_\perp$  remain invariant.

Initially ( $\mu \rightarrow \infty$ ) the oscillations are synchronized,  $\omega_c^\infty = \omega_{\text{synch}}$ , and all  $\mathbf{P}_\omega$  form a collective  $\mathbf{P}$ . As  $\mu$  decreases, the  $\mathbf{P}_\omega$  zenith angles spread out while remaining in a single co-rotating plane. In the end ( $\mu \rightarrow 0$ ) the co-rotation frequency is  $\omega_c^0$  and Eqs. (4) and (7) imply that all final  $\mathbf{H}_\omega$  and therefore all  $\mathbf{P}_\omega$  with  $\omega > \omega_c^0$  are aligned with  $\mathbf{B}$ , the others anti-aligned: a spectral split is inevitable with  $\omega_{\text{split}} \equiv \omega_c^0$  being the split frequency. The lengths  $P_\omega = |\mathbf{P}_\omega|$  are conserved and eventually all  $\mathbf{P}_\omega$  point in the  $\pm \mathbf{B}$  directions. Therefore the conservation of flavor-lepton number gives us  $\omega_{\text{split}}$ , for  $D_z > 0$ , by virtue of

$$D_z = \int_{-\infty}^0 P_\omega d\omega - \int_0^{\omega_{\text{split}}} P_\omega d\omega + \int_{\omega_{\text{split}}}^{+\infty} P_\omega d\omega. \quad (8)$$

In general,  $\omega_{\text{split}} = \omega_c^0 \neq \omega_c^\infty = \omega_{\text{synch}}$ .

For individual modes the EOMs given by  $\mathbf{H}_\omega$  are completely solved if we find  $\omega_c(\mu)$  and  $D_\perp(\mu)$ , the component transverse to  $\mathbf{B}$ , since  $D_z$  is conserved and given by the initial condition. From Eq. (4) we infer  $P_{\omega \perp}/P_\omega = H_{\omega \perp}/H_\omega$ , from Eq. (7)  $H_{\omega \perp} = \mu D_\perp$  and  $H_{\omega z} = \omega - \omega_c + \mu D_z$  so that

$$P_{\omega, z} = \frac{(\omega - \omega_c + \mu D_z) P_\omega}{\sqrt{(\omega - \omega_c + \mu D_z)^2 + (\mu D_\perp)^2}}, \quad (9)$$

$$P_{\omega \perp} = \frac{\mu D_\perp P_\omega}{\sqrt{(\omega - \omega_c + \mu D_z)^2 + (\mu D_\perp)^2}}. \quad (10)$$

Integration of the second equation over  $s_\omega d\omega$  gives us

$$1 = \int_{-\infty}^{+\infty} d\omega s_\omega \frac{P_\omega}{\sqrt{[(\omega - \omega_c)/\mu + D_z]^2 + D_\perp^2}}. \quad (11)$$

Projecting Eq. (5) on the  $x$ - $y$ -plane we find  $\omega_c = M_\perp/D_\perp(\mu)$  or explicitly

$$\omega_c = \frac{\int_{-\infty}^{+\infty} d\omega s_\omega \omega P_{\omega \perp}}{\int_{-\infty}^{+\infty} d\omega s_\omega P_{\omega \perp}} = \frac{\int_{-\infty}^{+\infty} d\omega s_\omega \omega P_{\omega \perp}}{D_\perp}. \quad (12)$$

For large  $\mu$  when the oscillations are synchronized, this agrees with the usual expression for  $\omega_{\text{synch}}$  [6], but it changes when the  $\mathbf{P}_\omega$  spread out in the zenith direction. Inserting Eq. (10) into Eq. (12) we find

$$\omega_c = \int_{-\infty}^{+\infty} d\omega s_\omega \frac{\omega P_\omega}{\sqrt{[(\omega - \omega_c)/\mu + D_z]^2 + D_\perp^2}}. \quad (13)$$

Given  $D_z$  and a spectrum  $P_\omega$ , we can determine  $\omega_c$  and  $D_\perp$  from Eqs. (11) and (13) for any  $\mu$ . These equations solve the EOMs explicitly in the adiabatic limit.

We have assumed that all  $\mathbf{P}_\omega$  are initially aligned. One can relax this restriction and allow some  $\mathbf{P}_\omega$  to have opposite orientation. If different species are emitted from a SN core with equal luminosities but different average energies, the spectra will cross over so that some range of modes is prepared, say, as  $\nu_e$  and another as  $\nu_x$ .

### IV. NEUTRINOS ONLY

We illustrate the power of our new results with a generic neutrino-only example ( $\mathbf{D} = \mathbf{P}$ ). The spectrum is taken box like with  $P_\omega = (2\omega_0)^{-1}$  for  $0 \leq \omega \leq 2\omega_0$  and 0 otherwise. With  $P_z$  being conserved we find from Eq. (8)

$$\omega_c = \omega_0 \times \begin{cases} 1 & \text{for } \mu \rightarrow \infty, \\ (1 - P_z) & \text{for } \mu \rightarrow 0. \end{cases} \quad (14)$$

The case  $P_z = 0$  is special because  $\omega_c = \omega_0$  remains fixed. For  $P_z = 1$  we have  $\omega_c^0 = 0$  and no flavor evolution. We use  $P_z = 0.5$  to show the initial and final  $P_z(\omega)$  in Fig. 1 (left). The dotted line denotes the adiabatic final

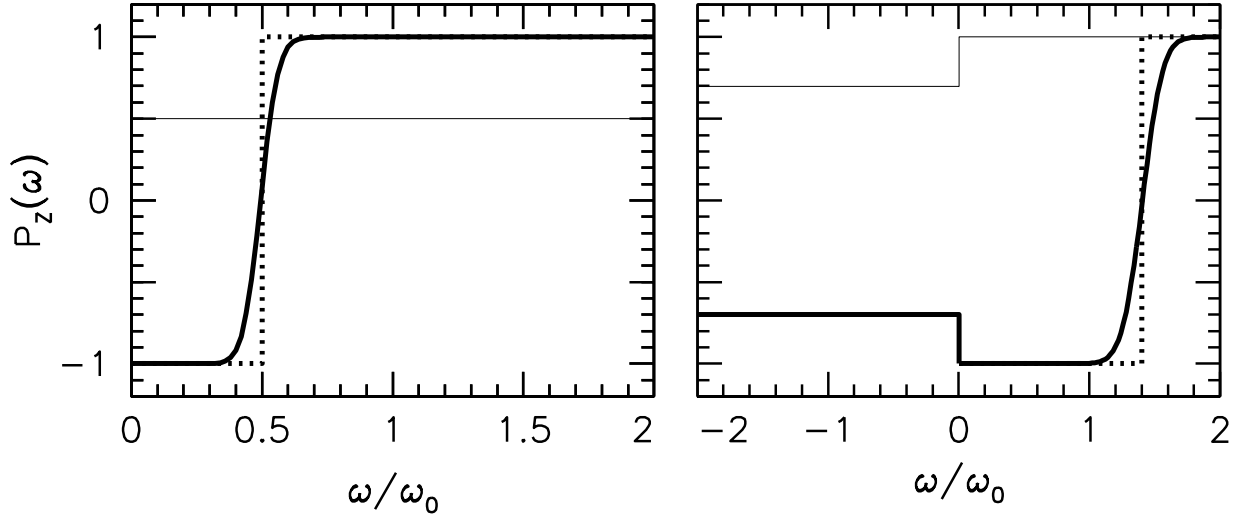


FIG. 1: Spectra of polarization vectors ( $z$ -component). Thin: initial. Thick: final. Dotted: fully adiabatic. Solid: numerical solution as described in the text. *Left*: Box-like initial  $\nu$  spectrum, large misalignment between  $\mathbf{B}$  and  $\mathbf{P}$ , and no  $\bar{\nu}$ . *Right*: Box-like  $\nu$  and  $\bar{\nu}$  spectra, 30% fewer  $\bar{\nu}$ , small initial misalignment ( $\sin 2\theta = 0.05$ ), and inverted hierarchy.

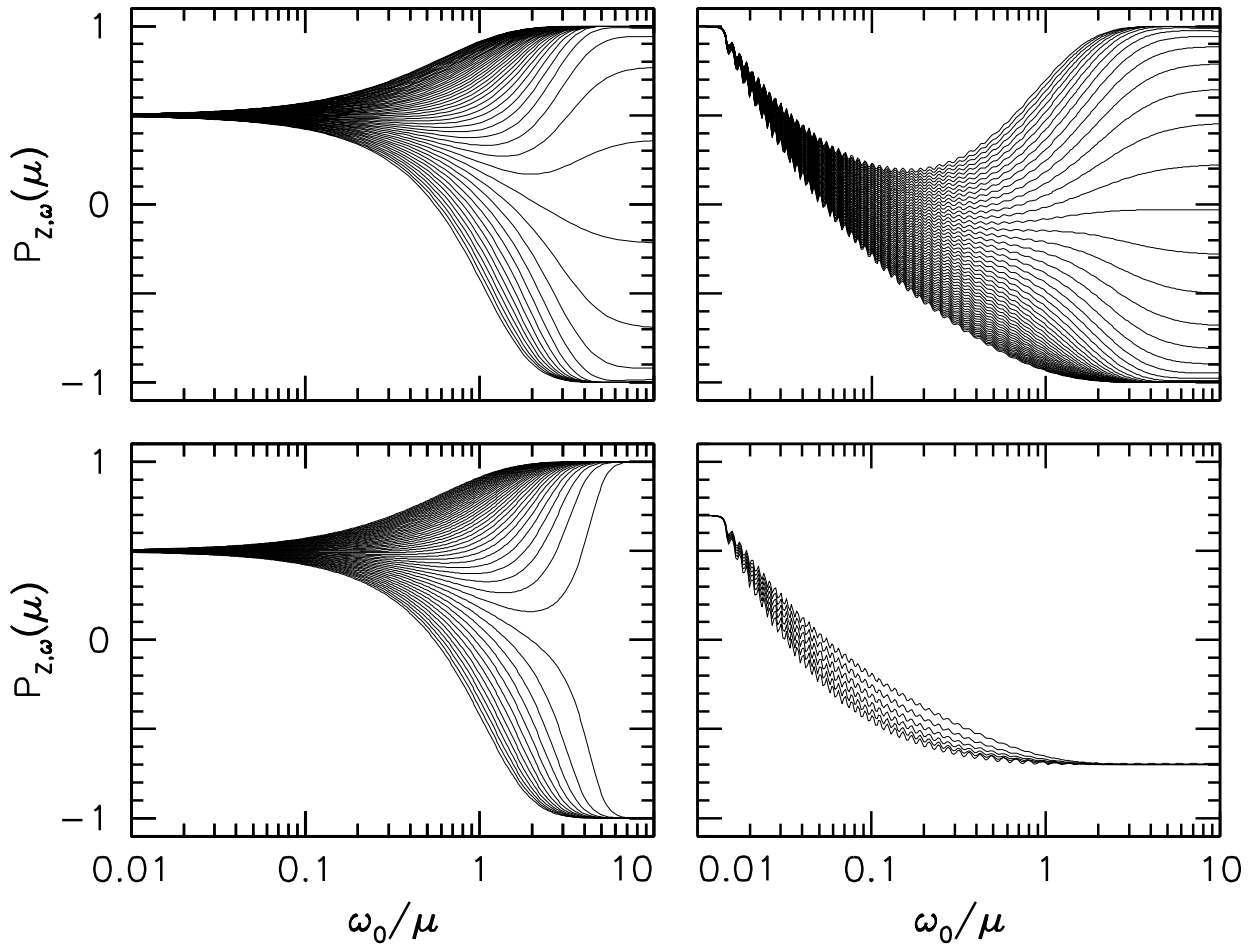


FIG. 2:  $P_{z,\omega}(\mu)$  for 51 modes. *Left*: Box-like  $\nu$ -only spectrum. Numerical solution of EOMs (*top*). Analytic adiabatic solution (*bottom*). *Right*: Box-like  $\nu$  and  $\bar{\nu}$  spectra. Numerical solution for  $\nu$  (*top*) and  $\bar{\nu}$  (*bottom*), here only 6 modes.

state where  $\omega_{\text{split}} = 0.5\omega_0$ . The solid line is from a numerical solution of the EOMs with  $\mu(t) = \mu_0 \exp(-t/\tau)$  and  $\tau^{-1} = 0.03\omega_0$ , typical for a SN. We have checked numerically that the split indeed becomes sharper with increasing  $\tau$  and thus increasing adiabaticity.

In Fig. 2 we show  $P_{\omega,z}(\mu)$  for 51 individual modes. They start with the common value  $P_{\omega,z} = 0.5(2\omega_0)^{-1}$ . Later they spread and eventually split, some of them approaching +1 and the others -1. Some modes first move down and then turn around as  $\omega_c$  changes. A few modes do not reach  $\pm 1$  because of imperfect adiabaticity.

For the box spectrum the integrals Eqs. (11) and (13) are easily performed and one can extract

$$\begin{aligned}\omega_c &= \omega_0 + \omega_0 P_z \left( \frac{1}{\kappa} - \frac{e^\kappa + e^{-\kappa}}{e^\kappa - e^{-\kappa}} \right), \\ P_\perp &= \sqrt{1 - P_z^2} \frac{2\kappa}{e^\kappa - e^{-\kappa}},\end{aligned}\quad (15)$$

where  $\kappa \equiv \omega_0/\mu$ . For  $\mu \rightarrow \infty$  and  $\mu \rightarrow 0$  the limits of  $\omega_c$  agree with Eq. (14) from lepton-number conservation. For  $\mu \rightarrow \infty$  we obtain  $P_\perp = \sqrt{1 - P_z^2}$ , representing the initial condition  $P = 1$ , and for  $\mu \rightarrow 0$  we find  $P_\perp = 0$ .

With Eq. (9) these results provide analytic solutions for the adiabatic  $P_{\omega,z}(\mu)$ . We show examples in Fig. 2 (bottom left) for comparison with the numerical solution of the EOMs. The agreement is striking and confirms the picture of adiabatic evolution in the co-rotating plane. The agreement is poor for modes close to the split ( $\omega \approx \omega_c^0$ ) at low neutrino densities ( $\mu < \omega_0$ ) where the evolution becomes nonadiabatic.

## V. ADIABATICITY CONDITION

The speed for the  $\mathbf{H}_\omega$  evolution in the co-rotating plane is  $d\theta_\omega/dt$ , where  $\cos\theta_\omega \equiv H_{\omega\perp}/H_\omega$ , while  $\mathbf{P}_\omega$  precesses with speed  $H_\omega$ . The evolution is adiabatic if the adiabaticity parameter  $\gamma_\omega \equiv |d\theta_\omega/dt| H_\omega^{-1} \ll 1$ . With Eqs. (7) and (10) we find

$$\gamma_\omega = \frac{\left( \frac{\omega - \omega_c}{\mu} + D_z \right) \frac{dD_\perp}{d\mu} + \frac{D_\perp}{\mu} \frac{d\omega_c}{d\mu} + D_\perp \frac{\omega - \omega_c}{\mu^2}}{\tau_\mu \left[ \left( \frac{\omega - \omega_c}{\mu} + D_z \right)^2 + D_\perp^2 \right]^{3/2}}, \quad (16)$$

where  $\tau_\mu \equiv |d \ln \mu / dt|^{-1}$ .

For our neutrino-only ( $D_\perp \rightarrow P_\perp$ ) box spectrum Eqs. (15) give  $dP_\perp/d\mu = -P_\perp(\omega_c - \omega_0)/\mu^2$  and  $d\omega_c/d\mu = P_z[1 - 4\kappa^2/(e^\kappa - e^{-\kappa})^2]$ . For  $\mu \gg \omega_0$  we obtain  $dP_\perp/d\mu \sim \omega_0^2/\mu^3$  and  $d\omega_c/d\mu \sim \omega_0^2/\mu^2$  so that the last term in the numerator of Eq. (16) dominates:  $\gamma_\omega \sim P_\perp(\omega - \omega_c)/(h_\mu \mu^2)$ . With  $\mu$  decreasing,  $\gamma_\omega$  increases and at  $\mu \sim \omega_0$  when  $\gamma_\omega \sim 1$ , adiabaticity violation begins. For  $\mu < \omega_0$  the denominator of Eq. (16) gives the dependence  $\gamma_\omega \propto (\omega - \omega_c)^{-3}$ , and therefore the closer  $\omega$  to  $\omega_c$  the stronger the adiabaticity violation.

## VI. INCLUDING ANTINEUTRINOS.

As a second generic case we now add antineutrinos. One important difference is that even a very small initial misalignment between  $\mathbf{D}$  and  $\mathbf{B}$  is enough to cause a strong effect. Consider a single energy mode for  $\nu$  with  $P = 1$  and one for  $\bar{\nu}$  with  $\bar{P} = \alpha < 1$  that are initially aligned in the flavor direction, now taken very close to the mass direction, and assume an inverted hierarchy. From the dynamics of the flavor pendulum [11, 12] we know that in the end  $\bar{\mathbf{P}}$  is antialigned with  $\mathbf{B}$ , whereas  $\mathbf{P}$  retains a large transverse component because  $P_z - \bar{P}_z$  is conserved: The system prepares itself for a spectral split.

Assuming box spectra for both  $\nu$  and  $\bar{\nu}$ , we show the initial and final  $P_{z,\omega}$  in Fig. 1 (right), for the inverted hierarchy,  $\sin 2\theta = 0.05$ , and  $\alpha = 0.7$ . From Eq. (12) one infers  $\omega_c^\infty = \omega_{\text{synch}} = \omega_0(1 + \alpha)/(1 - \alpha)$ . For  $\alpha = \frac{7}{10}$  this is  $\omega_c^\infty = \frac{17}{3}\omega_0 > 2\omega_0$ . Therefore, all modes have negative frequencies in the co-rotating frame and tilt away from  $\mathbf{B}$  (see also the numerical  $P_{\omega,z}$  in Fig. 2). The final split frequency is found from flavor lepton number conservation to be  $\omega_{\text{split}} = \omega_0(1 - D_z + \alpha) \approx \omega_0 2\alpha$ , using  $D_z \approx 1 - \alpha$  for  $\sin 2\theta \ll 1$ . With  $\alpha = \frac{7}{10}$  we find  $\omega_{\text{split}} = \frac{14}{10}\omega_0$  in agreement with Fig. 1. For  $0 < \alpha < 1$  we have  $0 < \omega_{\text{split}} < 2\omega_0$  so that the final split always occurs among the neutrinos. According to Fig. 2 the split starts when the vector  $\mathbf{D}$  develops a significant transverse component, and it proceeds efficiently in a region  $\mu \sim \omega_0$ .

The “wiggles” in the curves in the right panels of Fig. 2 stem from the nutation of the flavor pendulum [11, 12]. We have chosen a relatively fast  $\mu(t)$  evolution ( $\tau^{-1} = 0.1\omega_0$ ), implying poor adiabaticity, to avoid too many nutation periods on the plot. For a very slow  $\mu(t)$  the nutations disappear and the co-rotating frame removes the full global evolution of the system.

## VII. DISCUSSION

We have studied the phenomenon of spectral splits that is caused by neutrino-neutrino refraction in the SN dense-neutrino region. We have carried previous explanations of this novel effect [9, 12] to the point of explicit solutions in the adiabatic limit.

A spectral split occurs when a neutrino ensemble is prepared such that the common direction of the flavor polarization vectors deviates from the mass direction. An adiabatic density decrease turns all modes below a split energy  $E_{\text{split}} \equiv \Delta m^2/2\omega_{\text{split}}$  into the mass direction, and the others in the opposite direction. Remarkably, during this phase all modes remain in a single rotating plane, even after losing full synchronization.  $E_{\text{split}}$  is determined by lepton number conservation in the mass basis.

The spectral split is a generic feature of the adiabatic evolution when the density changes from large to small values. It can appear even in the absence of neutrino-neutrino interactions. Indeed, in the usual MSW case the evolution to zero density transforms  $\nu_e$  to  $\nu_2$  and

$\bar{\nu}_e$  to  $\bar{\nu}_1$  for all energies. This corresponds to  $\omega_{\text{split}} = 0$ . The neutrino-neutrino interactions shift  $\omega_{\text{split}}$  to non-zero values.

A spectral split is caused in the SN neutrino (but not antineutrino) flux by neutrino-neutrino interactions alone, especially during the accretion phase when ordinary MSW resonances occur far outside the dense-neutrino region. Later the matter profile may become so shallow that the H-resonance moves into this region [9, 10, 12]. The simultaneous action of collective effects and an ordinary MSW resonance may then cause spectral splits for both neutrinos and antineutrinos, leading to a rich phenomenology, perhaps modifying r-process nucleosynthesis [9, 12]. Of course, the fluxes will be further processed by ordinary conversion in the SN envelope [17, 18], thus modifying observable signatures. Still, observing spectral splits would provide a smoking gun signature both for the relevant neutrino properties and, if it occurs among antineutrinos at late times, for the occurrence of a shallow density profile above the neutrino sphere.

The neutrino flux emitted by a SN is anisotropic so that neutrinos on different trajectories experience different neutrino-neutrino interaction histories [9, 12] that would be expected to cause kinematical flavor decoherence of different angular modes [14]. A numerical ex-

ploration reveals, however, that in a typical SN scenario the deleptonization flux suppresses decoherence and the evolution is almost identical to that of an isotropic ensemble [15]. Our treatment of the spectral evolution is apparently applicable in a realistic SN context.

Collective neutrino oscillation phenomena in a SN may well be important for the explosion mechanism, r-process nucleosynthesis and may provide detectable signatures in a high-statistics signal from the next galactic SN. Building on previous ideas, our formalism gives a simple, elegant and quantitative explanation of seemingly impenetrable numerical results. Our approach provides the basis for developing a quantitative understanding of realistic consequences of collective neutrino oscillations for SN physics and observational signatures.

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